Unit 01 : SPUR GEAR

* Gear Drives :-

Gear drives are defined as tooth wheels which transmit power and motion from one shaft to another by means of successive engagement of teeth.

i) positive drive, constant velocity ratio.
ii) centre distance is small, compact construction
iii) transmit large power
iv) transmit motion even at low speed.
v) high efficiency (upto 99 %.)
vi) changing velocity ratio over wide range.

Disadvantages :- costly, high maintenance, require precise alignment of shaft.

* Classification of Gears :-

Spur, Helical, Bevel & worm wheels.

Spur → parallel shaft, impose radial load, involute teeth.

Helical → parallel shaft, involute profile in plane ⊥ to tooth element
- Helix angle of pinion & gear should be same but the hand of helix is opposite.
- imposes radial & thrust load
- Herringbone (Two gear as single unit but having opposite hand of helix ← no thrust)

Bevel :- shape of truncated cone
- gear tooth thickness & height decreases towards apex of cone.
- generally used for right angle shaft or near to 90°.
- straight or spiral bevel gear.
- impose radial & thrust load.

Worm Gear :- worm (screw) & worm wheel → impose high thrust load.
single or multistart with small lead.
- shaft whose axis not intersect & are perpendicular to each other.

* Selection of type of Gears :-

Factors considered while selecting gear, →

- general layout of shaft,
- speed reduction,
- power to be transmitted
- input speed
- cost.

Spur & Helical – parallel shaft
- 6:1 to 10:1 (velocity ratio ↑ ⇒ size ↑).
- spur gears are noisy at high speed so helical gears are preferred as it having pt. contact initially then line contact but spur gears are cheapest

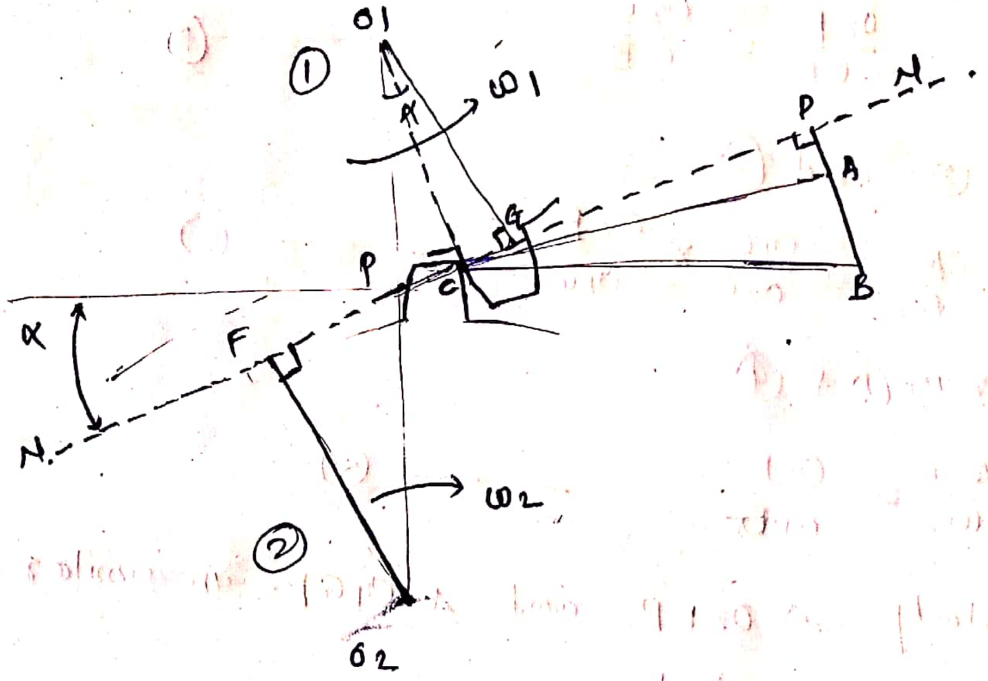
Bevel – intersecting shaft
- 1:1 to 3:1 (certain circumstances)

Worm Gear → axes of shafts are perpendicular & non intersecting
- 60:1 to 100:1
- Material handing equipments

Cross-Helical Gear – neither perpendicular nor intersecting shaft

* Fundamental law of Gearing:-

" It states that the common normal to the teeth profile at the point of contact should always pass through a fixed point called pitch point, in order to obtain a constant velocity ratio. "



O_1, O_2 = centres of two gears which are rotating with angular velocities ω_1 & ω_2 respt.

C = pt. of contact of tooth profile & NN is common normal at the point of contact.

$\overrightarrow{CA}$ is velocity of point C, when it is considered on gear 1 &
$\overrightarrow{CB}$ is velocity of point C, when it is considered on gear 2.

$CA \perp O_1C$ & $CB \perp O_2C$

- Projection of two vectors $\overline{CA}$ & $\overline{CB}$ i.e. CD along common normal NN must be equal otherwise teeth will not remain in contact & there will be a slip.

$v = r\omega$

$CA = \omega_1 \times O_1C$

$CB = \omega_2 \times O_2C$

$$\frac{\omega_1}{\omega_2} = \frac{O_2C}{O_1C} \times \frac{CA}{CB} \quad \text{——— (1)}$$

since $\Delta\, O_1CG$ and $\Delta\, CAD$ are similar

$$\frac{O_1C}{AC} = \frac{O_1G}{CD} \quad \text{———} \quad (2)$$

similarly, $\Delta\, O_2FC$ and CBD are similar

$$\frac{O_2F}{CD} = \frac{O_2C}{CB} \quad \text{———} \quad (3)$$

from (2) and (3)

$$\frac{CA}{CB} = \frac{O_1C}{O_2C} \times \frac{O_2F}{O_1G} \quad \text{———} \quad (4)$$

from (1) & (4)

$$\frac{\omega_1}{\omega_2} = \frac{O_2F}{O_1G} \quad \text{———} \quad (5)$$

similarly $\Delta\, O_2FP$ and $\Delta\, O_1GP$ are similar.

$$\frac{O_2F}{O_1G} = \frac{O_2P}{O_1P} \quad \text{———} \quad (6)$$

From (5) & (6)

$$\frac{\omega_1}{\omega_2} = \frac{O_2P}{O_1P}$$ & also $O_1P + O_2P = O_1O_2 = \text{constant}$

Therefore for a constant velocity ratio (ω_1/ω_2), P should be a fixed pt. This point is called pitch point.

It has been found that only involute & cycloid of curves satisfy the fundamental law of gearing.

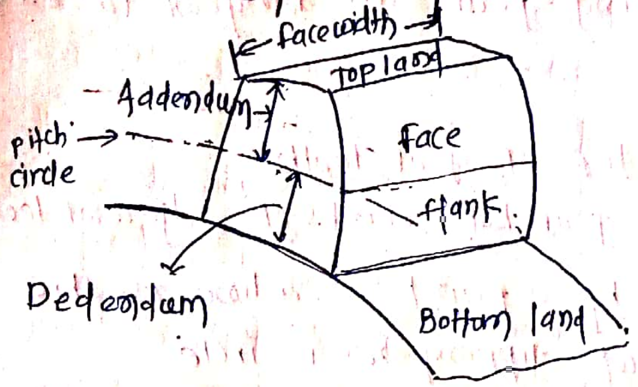
* Involute & Cycloidal Curve :-

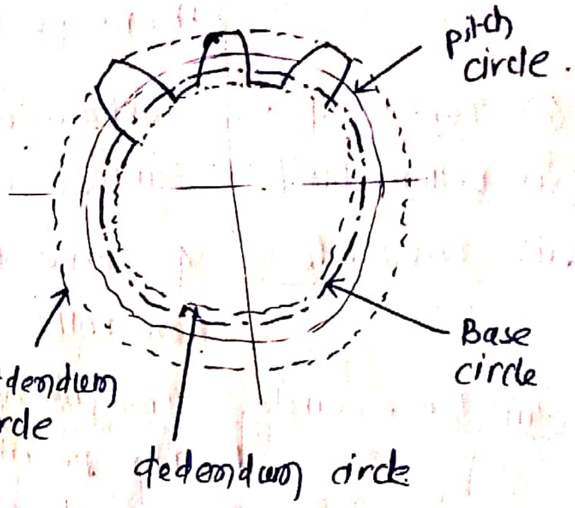
* Involute curve :- It is a curve traced by a point on a line as a line rolls without slipping on a circle

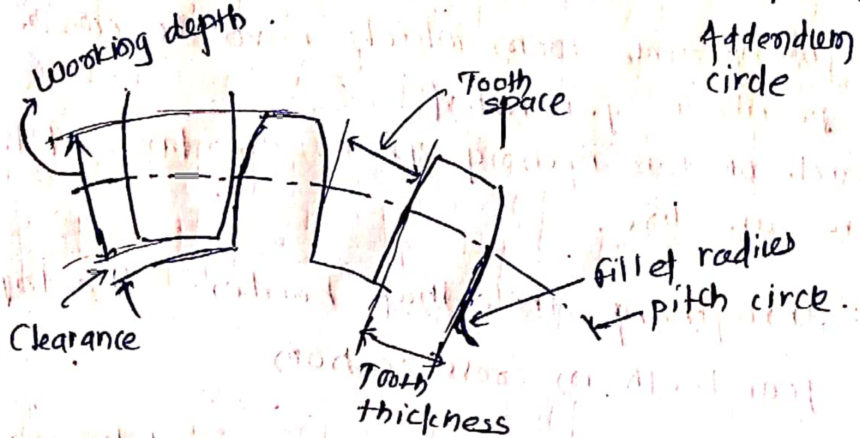
* Cycloid - It is a curve traced by a point on a circumference of a generating circle as it rolls without slipping along the inside and outside of another circle.

- Cycloidal profile consist of two curves - epicycloid & hypocycloid.
- Epicycloid - It is traced by a point on circumference of a generating circle as it rolls without slipping on the outside of pitch circle.
- Hypocycloid - It is traced by a pt. on the circumference of a generating circle as it rolls without slipping on inside of pitch circle.

* Terminology of Spur Gear :-







(i) Pinion :- smaller of the two mating gears.

(ii) Gear :- larger of two mating gears

(iii) Velocity Ratio / speed Ratio :- (i) Ratio of angular velocity of driving gear to the angular velocity of driven gear.

(iv) Transmission Ratio :- (i') = Ratio of angular velocity of first driving gear to angular velocity of last driven gear in gear train.

(v) Gear Ratio :- Ratio of no. of teeth on gear to that on pinion

$$i = \frac{Z_g}{Z_p} = \frac{n_p}{n_g}.$$

(vi) Pith surface :- imaginary planes, cylinder or cones that roll together without slipping.

(vii) Pith circle :- imaginary circle that rolls without slipping (curve of intersection of pitch surface of revolution)

(viii) Pith circle Diameter :- Diameter of pith circle., size of gear (pith diameter) (d) is generally specified by pith circle dia.

(ix) Pith Point :- Point on line of centers of two gears at which pith circle of mating gears are tangent to each other.

(x) Top land :- surface of the top of gear tooth.

(xi) Bottom land :- surface of the gear betⁿ flanks of adjacent teeth.

(xii) Involute :- A curve traced by a point on a line as the line rolls without slipping on a circle.

(xiii) Base Circle :- Imaginary circle from which involute curve of teeth profile is generated
- Base circle of two mating gears are tangent to the pressure line.

(xiv) Addendum Circle: Imaginary circle that borders the top of gear teeth in cross-section.

(xv) Addendum (ha) :- radial distance between pitch and addendum circle (height of tooth above pith circle

(xvi) Dedendum Circle :- Imaginary circle that borders the bottom (root circle) of spaces bet teeth in cross section.

(xvii) Dedendum (hf) :- radial distance betⁿ pitch & dedendum circle (depth below pitch circle)

(xviii) Clearance :- amount by which dedendum of given gear exceeds addendum of its mating gear.

(xix) face of tooth :- surface of gear tooth bet pitch cylinder and addendum cylinder

(xx) flank of tooth :- surface of gear tooth betⁿ pith cylinder and dedendum cylinder

(xxi) face Width :- Width of tooth measured parallel to axis

(xxii) Fillet Radius :- Radius that connect root circle to the profile of tooth.

(xxiii) circular tooth thickness :- length of the arc on pitch circle ~~sub~~tending a single gear tooth i.e. circular tooth thickness is half of circular pitch.

(xxiv) Tooth space :- Width of space betn two adjacent teeth measured along pitch circle

Tooth space = circular Tooth thickness = ½ circular pitch.

(xxv) Working Depth :- (h_k) depth of engagement of two gear teeth (sum of addendum of two mating gear)

(xxvi) Whole Depth (h) :- working depth + clearance or Addendum + dedendum of gear teeth.

* centre Distance : Distance betn centers of pitch circle of mating gear.

* Pressure angle :(α) angle which line of action makes with common tangent to pitch circle (angle of obliquity)

* line of action :- It is a common tangent to the base circle of mating gear.
- contact betn involute surfaces of mating gear teeth must be on this line to give smooth operation.

* Contact Ratio (m_p) :- no. of pair of teeth that are simultaneously engaged).
- for smooth power transfer, contact ration $\geq$ 1 (1.2)
- for industrial gear box (m_p > 1.6) ≈ 1.6 to 1.7.

* circular Pitch :- (p) distance measured along pitch circle betn two similar points on adjacent teeth.

$$p = \frac{\pi d}{z} \qquad z = \text{no. of teeth.}$$

* **Diameteral Pitch** :- ratio of no. of teeth to pitch circle. dia

$$P = (Z/d)$$

$$P \times p = \pi$$

* **Module** :- inverse of diametral pitch.

$$m = 1/P = d/z .$$

$$\boxed{d = mz}$$

Centre distance. $a = \frac{1}{2}(dp + dg) = \frac{1}{2}(mzp + mzg)$

$$a = m \frac{(zg + zp)}{2}$$

* Gear tooth failure :-

Two basic Modes of failure.

Breakage of tooth due to static & dynamic load

Surface distruction (Tooth wear)

Tooth Wear :-

i) Abrasive wear :- foreign particles in lubricant, dust, dirt etc scratch tooth surface.

- Remedies - use of oil filters, increasing surface hardness, & use of high viscosity fluid so thick lubricating film will be developed.

ii) Corrosive Wear :- corrosion due to corrosive elements such as ~~bio~~ extreme pressure additives in lubricant, & foreign particle due to contamination. causes uniform wear over entire surface

- Remedies - providing complete enclosure to avoid contamination, selecting proper additives & replacing lubricant over regular interval

iii) Initial Pitting :- small pits at high spots. causes due to error in tooth profile, surface irregularities, & misalignment

- Remedies - precise machining of gear, correct alignment & reducing dynamic load.

iv) Destructive Pitting :-

- when load on gear tooth exceeds surface endurance strength of material
- characterise by continuous growing pits & premature breakage of tooth.
- Destructive pitting depends on magnitude of Hertz contact stress & no. of stress cycle.

Remedies → Avoid improper designing of gear, increase surface hardness

(v) Scoring :- Excessive surface pressure, high surface speed & inadequate supply of lubricant result in breakdown of oil film & consequently heating of meshing teeth.

— scoring is stick-slip phenomenon at high spots

Remedies :- Avoid by proper surface speed, surface pressure & flow of lubricant.

— provide fins for proper cooling etc.

* Number of teeths :-

- As no. of teeth decreses, a point is reached when there is interference and standard tooth profile require modification.
- Minimum no. of teeth to avoid interference is given by

$$Z_{min} = \frac{2}{\sin^2\alpha} \qquad \alpha = \text{press. angle}$$

- In practice, giving a slight radius to the tip of tooth can further reduce the value of Z_{min}.

Pressure angle (α)	14.5°	20°	25
Z_{min} (theoretical)	32	17	11
Z_{min} (Practical	27	14	9

- for 20° full depth → Z_{min} on pinion = 18 to 20 for safe
- As no. of teeth increases, – PCD ↑ → size of gear ↑ → cost ↑

* Hunting tooth :- for uniform distribution of wear.

- Suppose $Z_p = 20$ & $Z_g = 40$ then after every two revolution of pinion, the same pair of teeth will engage.
- If however, we take $Z_p = 20$ & $Z_g = 41$, the pinion will rotate 41 times before same pair of teeth will engage. This extra tooth is called hunting tooth which evenly distribute wearing phenomenon.
- For this, it is advicable to sligthy alter velocity ratio

* In multistage gearbox, velocity ratio of each stage should not exceed 6:1. The intermediate speeds are arranged in geometric progression.

- reduction of each stage (i) is obtained from transmission ratio (i') as
  for 2 stage $i = \sqrt{i'}$
  for 3 stage $i = \sqrt[3]{i'}$

* Face Width :- (b)

- In lewis equation, P_t is uniformly distributed over entire face width.
- If face width is too large, there is a possibility of concentration of load at one end of gear tooth due to misalignment, elastic deformation of shaft or warping of gear tooth
- If face width is small, it has poor capacity to resist shock & absorb vibration so faster wear rate will be there. (Narrow face width results in coarse pitch)
- Optimum range of face width

$$8m < b < 12m$$

- In preliminary stage of gear design $b = 10m$.

Interference & undercutting :-

→ outside base circle profile is involute.
→ overlapping of tooth profiles & cutting each other - interference
→ Interference → due to non-conjugate action result in excessive wear, vibration & jamming
→ undercutting solves problem of interference but weaken the tooth. also removes portion of involute curve near base circle. so length of contact reduces.

method to eliminate interference :-

1) increase no. of teeth on pinion
— no. of teeth ↑ → gearbox size ↑ → pitch line velocity ↑.

14°.5 → 32 teeth
20° full depth → 17
20° stub → 14.

2) increase pressure angle :- result in smaller base circle so more portion of tooth profile is involute.

iii) use long & short Addendum Gearing :-
non-standard & non-interchangeable gear.
→ non standard & non-interchangeable gear.
→ best course to avoid interference is to avoid theoretical conditions that result in overlapping profile of mating gear.

* **Backlash** :- play in gear.
Advantage :- compensate thermal expansion.
compensate m/c errors.
- prevent from jamming. (rotate freely & smoothly)

Beam strength of Gear tooth:- Analysis of Bending stress in gear was done by Wilfred Lewis.

~~Assumption~~ :-

- In Lewis analysis, gear tooth is treated as cantilever beam.
- Tangential component (P_t) causes bending moment @ base of tooth.

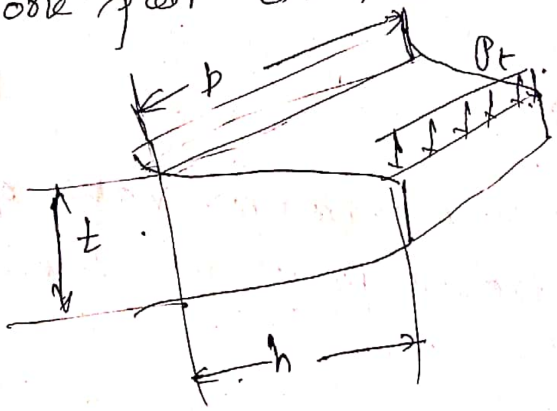
Assumption :-

i) P_r effect neglected (compressive stress)

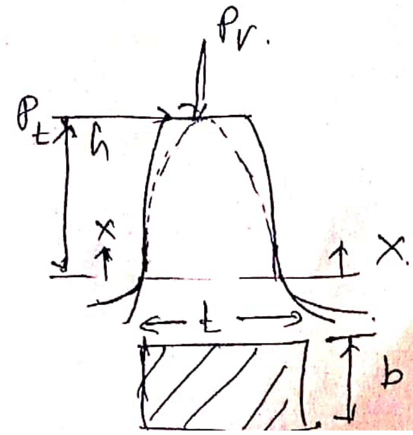
ii) P_t uniformly distributed over face width of gear. (gear is rigid & accurately m/c)

iii) stress concentrate — neglected.

iv) one pair of teeth in contact & take total load



- Cross section of beam varies from free end to fixed end. so parabola constructed within tooth profile &.
- The advantage of parabolic outline is that it is a beam of uniform strength.
- Weakest section of tooth at X-X where parabola is tangent to tooth profile.



At section XX

$M_b = P_t \times h \qquad I = \frac{1}{12} bt^3, \quad y = t/2$

$$\sigma_b = \frac{M_b \cdot y}{I} = \frac{(P_t \times h)(t/2)}{\left(\frac{1}{12} bt^3\right)}$$

Rearrange.

$$P_t = b \cdot \sigma_b \cdot \left(\frac{t^2}{6h}\right)$$

$$P_t = m \cdot b \cdot \sigma_b \left(\frac{t^2}{6hm}\right)$$

$$\boxed{P_t = m \cdot b \cdot \sigma_b \cdot Y} \qquad Y = \frac{t^2}{6hm} \Rightarrow \text{Lewis form factor.}$$

Relation betn P_t & σ_b.

Beam strength (S_b) :- It is maximum value of tangential force that tooth can transmit without bending failure.

$$\boxed{S_b = m \cdot b \cdot \sigma_b \, Y}$$

Beam strength > effective force betn meshing teeth

$$S_b \geq P_{eff}.$$

Effective load on Gear teeth:-

$$M_t = \frac{60 \times 10^6 (P_{kW})}{2\pi n}$$

$$P_t = \frac{2M_t}{d'}$$

→ P_t depends on rated power & ~~rated~~ rated speed

→ In practical, torque developed by source of power varies during work cycle.

— similarly torque required by driven m/c varies.

→ two side balanced by flywheel.

→ In gear design, maximum force is design criteria.

This is accounted by service factor. (C_s)

$$C_s = \frac{\text{max. torque}}{\text{rated torque}} = \frac{(P_t)_{max}}{P_t}$$

← depend on rated torque

$$(P_t)_{max} = C_s (P_t)$$

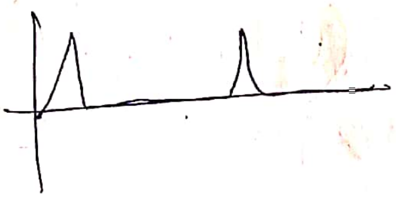
for electric Motor.

$$C_s = \frac{\text{starting torque}}{\text{rated torque}}$$

Driving	Driven: uniform	moderate shock	heavy shock
uniform	1	1.25	1.75
light shock	1.25	1.5	2
Medium shock	1.5	1.75	2.25

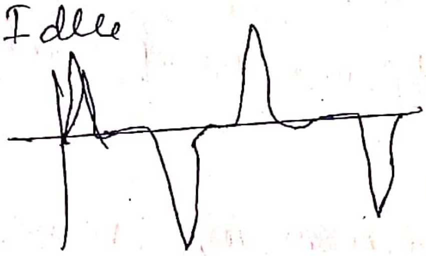
→ Driving & driven → repeated stress

$\sigma_m = \frac{1}{2}\sigma_{max}$



$\sigma_a = \frac{1}{2}\sigma_{max}$.

→ Idler



$\sigma_m = 0$

$\sigma_a = \sigma_{max}$.

B. Buckingham:

∴ Endurance surface limit of gear teeth is approximately 1/3 of UTS of material

$$\sigma_b = S_e = \frac{1}{3} S_{ut}$$

for bronze → 40 % UTS.

Two methods to account for dynamic load

i) Approximate estimation by velocity factor in preliminary stage of gear design

ii) precise calculation by Buckingham's equation in final stage of gear design.

- in preliminary stage, it is difficult to calculate dynamic load exactly.
- To overcome this, velocity factor C_v developed by Barth is used.

Velocity factor values:-

i) $C_v = \frac{3}{3+V}$ for ordinary & commercially cut gear made with form cutter & $V < 10$ m/s

ii) $C_v = \frac{6}{6+V}$ for accurately hobbed & generated gear with $V < 20$ m/s

iii) $C_v = \frac{5.6}{5.6+\sqrt{V}}$ for precision gear with shaving, grinding & lapping o/p $V > 20$ m/s.

V = pitch line velocity in m/s $\longrightarrow$ $V = \frac{\pi d n}{60 \times 10^3}$ m/s n = rpm

Effective load between two meshing teeth is given by

$$P_{eff} = \left(\frac{C_s}{C_v}\right) P_t$$

By velocity factor, dynamic load calculation has advantages

i) easier to calculate C_v & design gear

ii) gives satisfactory result

Disadvantages i) Dynamic load depends upon - mass of gear, mass connected to gear shaft, properties of gear material.

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velocity factor – neglect these factors

iii) use of velocity factor is restricted to a limited range of pitch line velocities. It is not possible to extrapolate the values.

- final stage of gear design, when gear dimensions are known, error specified & quality of gears determined.
- Dynamic load calculated by Buckingham equation.

- The effective load is given by

$$P_{eff} = (C_s P_t + P_d)$$

P_d = incremental dynamic load or additional dynamic load due to dynamic conditions betn two meshing teeth.

$$P_d = \frac{21V(Ceb + P_t)}{21V + \sqrt{(Ceb + P_t)}}$$

V = pitch line velocity m/s
C = Deformation factor N/mm²
e = sum of error betn two meshing teeth (mm)
b = face width of teeth mm
P_t = tangential force on rated torque N.

C :- Deformation factor
- depends on E of pinion & gear & form of tooth or pressure angle.

$$C = \frac{k}{\left(\frac{1}{E_p} + \frac{1}{E_g}\right)}$$

k = constant depending upon form of tooth.
E_p = N/mm² for pinion
E_g = N/mm² for gear.

- The values of k for various tooth forms.
  - k = 0.107 → 14.5° full depth teeth
  - k = 0.111 → 20° full depth teeth.
  - k = 0.115 → 20° stub teeth

for

→ Incremental dynamic load (P_d) calculated by Buckingham equation is far more than corresponding load calculated by velocity factor (3 to 4 times load (P_t) due to power transmission)

— In actual practice, actual dynamic load ~~calculated~~ is less than calculated values by Buckingham's eqn.

— This is because the equation is mainly applicable to large gears with connected masses ~~than~~ that rotate at moderate speeds.

— The actual dynamic load is less in following cases.

i) small gear transmitting low power
ii) high speed light load gears.
iii) small gear on small shaft
iv) gear shafts up to 50 mm dia.
v) small gears transmitting less than 15 kW of power

The error e is given by

$$e = e_p + e_g.$$

e_p = error in pinion
e_g = error in gear.

— error depends upon quality of gear & method of manufacture.

$$\phi = m + 0.25\sqrt{d'}$$

ϕ = tolerance factor, m = module.
d' = pitch circle dia. (mm).

Estimation of module Based on Beam strength:-

In order to avoid failure of gear tooth due to bending.

$$S_b > P_{eff}.$$

Introducing FoS

$$S_b = P_{eff}.(FS) \quad \text{——— (1)}$$

Recommended fos → 1.5 to 2

Tangential component is given by.

$$P_t = \frac{2M_t}{d'} = \frac{2M_t}{mz} = \frac{2}{mz}\left[\frac{60\times10^6\,(P_{kw})}{2\pi n}\right]$$

$$P_{eff} = \frac{C_s}{C_v} P_t = \frac{C_s}{C_v}\cdot\frac{2}{mz} \times \frac{60\times10^6\,(P_{kw})}{2\pi n}. \quad \text{———}$$

$$P_{eff} = \frac{60\times10^6}{\pi}\left(\frac{P_{kW}}{mzn}\cdot\frac{C_s}{C_v}\right) \quad \text{——— (2)}$$

Beam strength eqn

$$S_b = m\,b\cdot\sigma_b\cdot Y. = m^2\left(\frac{b}{m}\right)\cdot\left(\frac{S_{ut}}{3}\right)\cdot Y \quad \text{——— (3)}$$

From (1), (2) & (3)

$$m = \left[\frac{60\times10^6}{\pi}\left\{\frac{P_{kW}\cdot C_s\cdot fs}{zn\,C_v\left(\frac{b}{m}\right)\left(\frac{S_{ut}}{3}\right)Y}\right\}\right]^{1/3}.$$

The above eqn is used in preliminary stage of gear design

Wear strength of Gear tooth:- Buckingham eq^n.

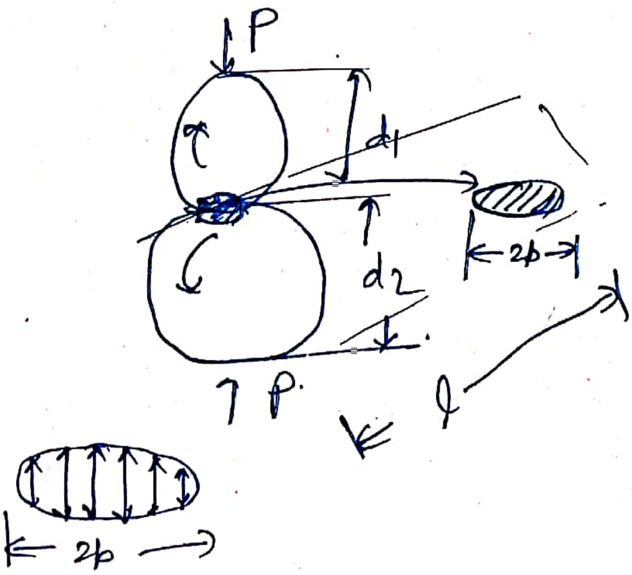
- failure of gear due to pitting occurs when contact stress bet^n two teeth > surface endurance strength of material.

- Pitting - surface fatigue failure
- In order to avoid pitting failure surface hardness should be selected in such a way that wear strength of gear tooth is more than effective load bet^n meshing teeth.

- Buckingham's eq^n is based on Hertz theory of contact stress. When two cylinder pressed together, the contact stresses are given by

$$\sigma_c = \frac{2P}{\pi b l} \quad \text{—— (a)}$$

$$b = \sqrt{\frac{2P(1-\mu^2)\left(\frac{1}{E_1}+\frac{1}{E_2}\right)}{\pi l\left(\frac{1}{d_1}+\frac{1}{d_2}\right)}} \quad \text{—— (b)}$$



μ = pissons ratio
E_1, E_2 = moduli of elasticity
d_1, d_2 = dia. of two cylinders
l = axial length of cylinder

- Due to deformation under action of load P, a rectangular surface of width (2b) & length (l) is formed bet^n two cylinders.

- Elliptical stress distribution.

- squaring (b) & putting in (a) gives

$$\sigma_c^2 = \frac{1}{\pi(1-\mu^2)}\left(\frac{P}{l}\right)\left[\frac{\frac{1}{r_1}+\frac{1}{r_2}}{\frac{1}{E_1}+\frac{1}{E_2}}\right] \quad \text{--- (c)}$$

r_1 & r_2 = radii of two cylinder

Put
$\mu = 0.3$

$$\sigma_c^2 = 0.35\left(\frac{P}{l}\right)\left[\frac{\frac{1}{r_1}+\frac{1}{r_2}}{\frac{1}{E_1}+\frac{1}{E_2}}\right] \quad \text{--- (d)}$$

The above eq^n is based on assumptions.

i) cylinder made of isotropic material
ii) elastic limit is not exceeded
iii) r_1 & r_2 are very large as compare to $(2b)$ width of deformation.

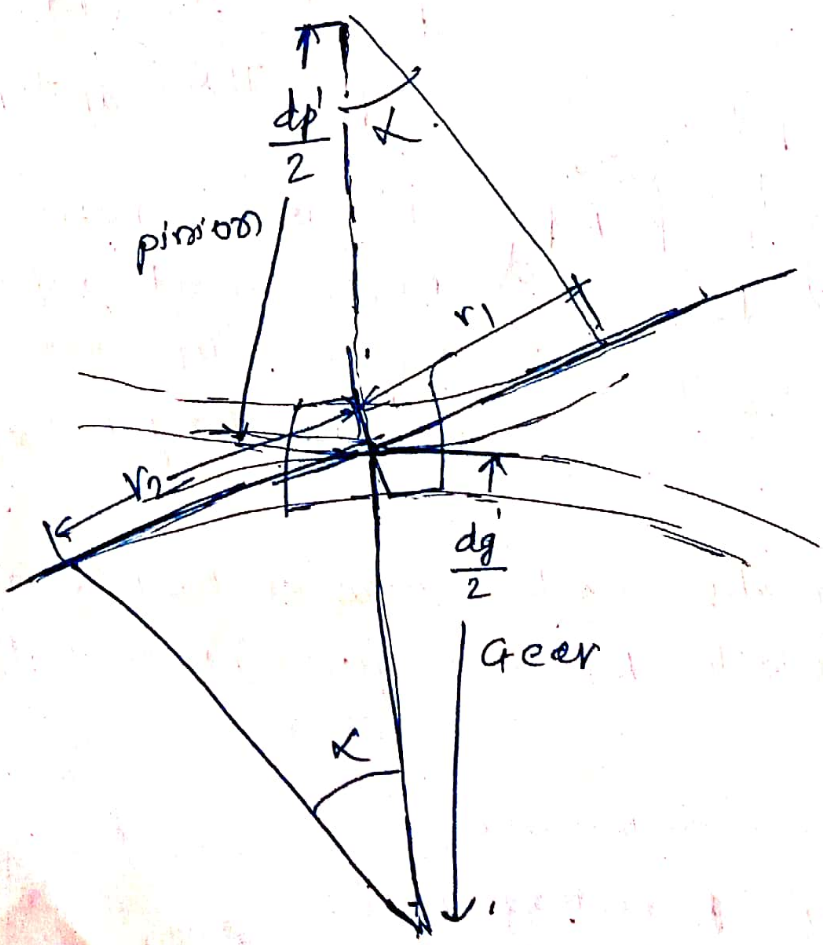


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figure shows the contact betn two meshing teeth at pitch point. The radii r_1 & r_2 are replaced by radii of curvature at pitch point.

$$\sin\alpha = \frac{r_1}{(d_p'/2)} \qquad \sin\alpha = \frac{r_2}{(d_g'/2)}$$

$$r_1 = \frac{d_p' \sin\alpha}{2} \qquad r_2 = \frac{d_g' \sin\alpha}{2}$$

→ There are two reasons for taking radii of curvature at pitch point.

- Wear on gear tooth generally occurs at or near pitch line. When only one pair of teeth carries entire load.
- When contact takes place at top or bottom of tooth profile, usually 2 pair of meshing teeth share the load.
- Dynamic load imposed on gear tooth near pitch line area
- Therefore it is more reasonable to select radii of curvature at pitch point.

$$\left(\frac{1}{r_1} + \frac{1}{r_2}\right) = \frac{2}{\sin\alpha}\left(\frac{1}{d_p'} + \frac{1}{d_g'}\right) \quad \text{——— (e)}$$

A ratio factor $\varphi = \frac{2Z_g}{Z_g + Z_p} = \frac{2d_g'}{d_g' + d_p'}$ $(d = mz)$ ——— (f)

$$\left(\frac{1}{d_p'} + \frac{1}{d_g'}\right) = \left(\frac{d_p' + d_g'}{d_p' d_g'}\right) = \frac{2}{\varphi\, d_p'} \quad \text{——— (g)}$$

$$\text{so} \left(\frac{1}{r_1} + \frac{1}{r_2}\right) = \frac{2}{\sin\alpha} \times \frac{2}{\varphi \cdot d_p'} = \frac{4}{\varphi\, d_p' \cdot \sin\alpha} \quad \text{——— (h)}$$

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force acting along pitch line

$$P = P_N = \frac{P_t}{\cos\alpha} \quad \text{——— (j)}$$

- Axial length of gears is face width b.

$$l = b \quad \text{——— (f)}$$

substituting (h), (j) & (k) in (g)

$$\sigma_c^2 = 0.35 \cdot \frac{P_t}{\cos\alpha} \times \frac{1}{l} \times \frac{\frac{4}{Q \cdot d_p' \sin\alpha}}{\left(\frac{1}{E_1} + \frac{1}{E_2}\right)}$$

$$\sigma_c^2 = \frac{1.4 \cdot P_t}{b\, Q \cdot d_p' \sin\alpha \cdot \cos\alpha \left(\frac{1}{E_1} + \frac{1}{E_2}\right)}$$

A load stress factor k is defined as.

$$k = \frac{\sigma_c^2 \cdot \sin\alpha \cos\alpha \left(\frac{1}{E_1} + \frac{1}{E_2}\right)}{1.4}$$

$$\therefore \frac{\sigma_c^2 \sin\alpha \cos\alpha \left(\frac{1}{E_1} + \frac{1}{E_2}\right)}{1.4} = \frac{P_t}{b\,Q \cdot d_p'}$$

$$k \cdot b \cdot Q \cdot d_p' = P_t$$

$$\boxed{P_t = b\,Q \cdot d_p'\, k} \longrightarrow \text{relation bet}^n\ P_t\ \&\ \sigma_c \text{ contact stress } (k)$$

$P_t \uparrow \Rightarrow \sigma_c \uparrow$ contact stress

- Pitting occur when contact stress reaches magnitude of surface endurance limit
- Wear strength is a max. value of tangential force that tooth can transmit without pitting failure

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$$\boxed{S_w = b\, Q\, d_p'\, K}$$ → Buckingham's eq^n for wear.

S_w = Wear strength of gear tooth.

σ_c = surface endurance strength of material.

Q for internal gear $Q = \frac{2 Z_g}{Z_g + Z_p}$.

* Expression for load stress factor K

If both gears are made of steel with 20° pressure angle

$E_1 = E_2 = 206 \times 10^3 \ N/mm^2$

$\alpha = 20$

According to G. Niemann

$\sigma_c = 0.75 (BHN) \ kgf/mm^2$

$= 0.75 (BHN)(9.81) \ N/mm^2$

BHN → Brinell Hardness no.

$$K = \frac{\sigma_c^2 . \sin\alpha . \cos\alpha . \left(\frac{1}{E_1} + \frac{1}{E_2}\right)}{1.4}$$

$$= \frac{0.27^2 \times 9.81^2 \times BHN^2 \times \sin(20) \cos 20 \left(\frac{2}{206000}\right)}{1.4}$$

$$K = 0.156 \left(\frac{BHN}{100}\right)^2$$

Estimation of module Based on Wear strength

- To avoid failure of gear tooth due to pitting

$$S_w > P_{eff}.$$

$$S_w = P_{eff} \cdot (fs) \quad \text{——— (a)} \qquad fs \rightarrow 1.5 \text{ to } 2$$

$$P_{eff} = \frac{60 \times 10^6}{\pi} \left(\frac{P_{kw}}{m z n} \times \frac{C_s}{C_v} \right) \quad \text{——— (b)}$$

$$S_w = b \, Q . d_p' \, K = m \left(b/m \right) Q \left(m z_p \right) K)$$

$$S_w = m^2 \left(b/m \right) Q \cdot z_p \cdot K \quad \text{——— (c)}$$

substituting (b) & (c) in (a)

$$\left[m = \frac{60 \times 10^6}{\pi} \left\{ \frac{P_{kw} \cdot C_s \cdot (fs)}{z_p^2 n_p C_v \left(b/m \right) Q K} \right\} \right]$$

Example

It is required to design a pair of spur gear with 20° full depth involute teeth consisting of a 20 teeth pinion meshing with a 50 teeth gear. The pinion shaft is connected to a 22.5 kW, 1450 rpm electric motor. The starting torque of the motor can be taken as 150 % of the rated torque. The material for the pinion is Plain Carbon steel Fe 410 (S_{ut} = 410 N/mm²) while the gear is made of grey cast iron FG 200 (S_{ut} = 200 N/mm²). The factor of safety is 1.5. Design the gears based on Lewis equation and using velocity factor to account for the dynamic load.

→ Given:-

$\alpha = 20°$ | $C_s = 1.5$

$Z_p = 20 \rightarrow Y = 0.32$ | starting torque = 150 % rated torque.

$Z_g = 50 \rightarrow Y = 0.408$ | $S_{ut} = 410\ N/mm^2$ — pinion.

$P_{kW} = 22.5$ kW | $S_{ut} = 200\ N/mm^2$ — Gear

$n_p = 1450$ rpm | $fs = 1.5$

- Deciding weaker bet^n pinion & gear.

$$(\sigma_b \cdot Y)_{pinion} = \left(\frac{S_{ut}}{3}\right) \cdot Y \qquad Z_p = 20 \Rightarrow Y = 0.32$$

$$= \left(\frac{410}{3}\right)(0.32)$$

$$= 43.7333 \quad \text{———} \quad (1)$$

$$(\sigma_b \cdot Y)_{Gear} = \left(\frac{S_{ut}}{3}\right) Y = \left(\frac{200}{3}\right)(0.408) = 27.2 \quad \text{—} \quad (2)$$

from (1) & (2), gear is weakest so design for the Gear.

- Design based on Lewis eq^n & using velocity factor for dynamic load.

$$m = \sqrt[3]{\frac{60 \times 10^6}{\pi}\left(\frac{P_{kW} \cdot C_s \cdot fs}{\cancel{n}\, Z\, C_v \left(\frac{b}{m}\right)\left(\frac{S_{ut}}{3}\right) Y}\right)}$$

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$$i = \frac{Z_g}{Z_p} = \frac{n_p}{n_g}.$$

$$n_g = \frac{Z_p}{Z_g} n_p = \frac{20}{50}(1450) = 580.$$

Assuming n_g: $V = 5 m/s$ $\quad C_v = \frac{3}{3+V} = 3/8$

$$m = \sqrt[3]{\frac{60 \times 10^6}{\pi} \cdot \frac{22.5 \times 1.5 \times 1.5}{50 \times 580 \times \frac{3}{8} \times 10 \times \frac{200}{3} \times 0.408}}$$

$m = 6.89$ mm $\simeq 7$ mm.

$d_p = m z_p$

$d_g = m z_g =$

$b = 10m =$

Check for Design :-

$$M_t = \frac{60 \times 10^6 \, P_{kW}}{2\pi n_g} = \frac{60 \times 10^6 \times 22.5}{2\pi \times 580} = 370446.85 \text{ Nmm}$$

$$P_t = \frac{2M_t}{d_g} = \frac{2(370446.85)}{350} = 2116.84 \text{ N}$$

$$V = \frac{\pi d_p n_p}{60 \times 10^6} = \frac{\pi(140)(1450)}{60 \times 10^3} = 10.63 \text{ m/s} \Rightarrow C_v = \frac{6}{6+V} = 0.3608$$

$$P_{eff} = \frac{C_s}{C_v} P_t = \frac{1.5}{0.3608} \times 2116.84 = 8800.61 \text{ N}$$

$$S_b = m b \sigma_b Y = 7(70)\left(\frac{200}{3}\right)(0.408) = 13328 \text{ N}.$$

$$FS = \frac{S_b}{P_{eff}} = \frac{13328}{8800.61} = 1.51 \;\; > \;\; 1.5.$$

The design is safe.

Example :-

It is required to design a two-stage spur gear reduction unit with 20° full depth involute teeth. the input shaft rotates at 1440 rpm and receives 10 kW power through a flexible coupling. the speed of the output shaft should be approximately 180 rpm. The gears are made of plain carbon steel 45C8 (S_{ut} = 700 N/mm²) and heat-treated to a surface hardness of 340 BHN. the gears are to be machined to the requirement of grade 6. The service factor can be taken as 1.5.

(i) Assuming that the dynamic load to be proportional to the pitch line velocity, estimate the required value of the module. The factor of safety is 1.5.

(ii) select the first preference value of the module and determine the correct value of factor of safety for bending using Buckingham's equation.

(iii) Determine the factor of safety against pitting

(iv) Give a list of gear dimensions.

→ Given :-

$\alpha = 20°$ | Grade 6 ⇒ $e = 8 + 0.63\phi$

$n_p = 1440$ rpm | $C_s = 1.5$

$P_{kW} = 10$ kW.

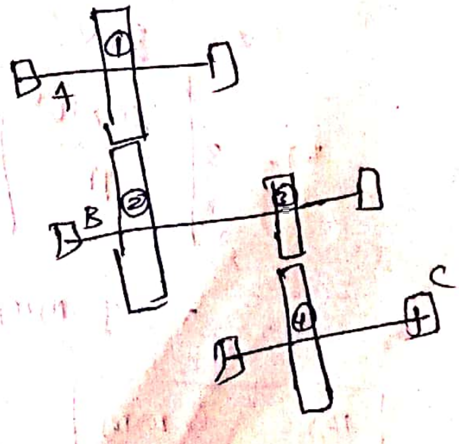
$n_p = 180$ rpm. = output shaft

$S_{ut} = 700$ N/mm²

BHN = 340.

$$i' = \frac{1440}{180} = 8$$

$$i = \sqrt{i'} = \sqrt{8} = 2.8284$$



- for 20° pressure angle and having same ~~no. of~~ material, pinion is the weakest & no. of teeth on pinion are

$Z_p = 18$

$Z_g = i \cdot Z_p = 2.8284 (18) = 50.91 \approx 51$

- for ease of manufacturing, gear 1 & (3) are identical & gear 2 & (4) are identical so

$Z_1 = Z_3 = 18$ & $Z_2 = Z_4 = 51$

- speed of the shafts are as follows

$n_A = 1440$ rpm.

$i = \frac{n_A}{n_B}$ $\quad n_B = (n_A / i) = (1440 / 2.8284)$

$n_B = 509.12$ rpm

$i = \frac{n_B}{n_C}$ $\quad n_C = \frac{n_B}{i} = \frac{509.12}{2.8284} = 180$ rpm.

- The pinion and gear of 2nd stage are subjected to more torque than pair 1. so - design for 2nd stage gear.

for $Z_p = 18 \longrightarrow Y = 0.308$

$V = 5$ m/s Assuming

$C_v = \frac{3}{3+V} = 3/8.$

$$m = \sqrt[3]{\frac{60 \times 10^6}{\pi} \left\{ \frac{P_{kW}\ C_s \cdot fs}{Z_3\, n_3\, C_v \left(\frac{b}{m}\right)\left(\frac{S_{ut}}{3}\right) Y} \right.}$$

$= 5.59$ mm ≈ 6 mm

$\boxed{m = 6 \text{ mm}}$

For FS using Buckingham's eqn.

$$S_b = P_{eff} \cdot FS$$

$$P_{eff} = C_s \cdot P_t + P_d \qquad P_d = \frac{21V(Ceb + P_t)}{21V + \sqrt{Ceb + P_t}}$$

$$S_b = m b \sigma_b \cdot Y = 6 \times 60 \left(\frac{700}{3}\right)(0.308) = 25872\ N.$$

$$P_t = \frac{2M_t}{d_3} = \frac{2}{m z_3}\left(\frac{60 \times 10^6\ P_{kW}}{2\pi n_3}\right) = 3479.47\ N.$$

For Dynamic load

For Grade 6, $e = 8 + 0.63\phi$

$$e_p = 8 + 0.63\left(m + 0.25\sqrt{108}\right) = 13.41\ \mu m$$

$$e_g = 8 + 0.63\left(m + 0.25\sqrt{306}\right) = 14.53\ \mu m$$

$$e = e_p + e_g = 27.95\ \mu m = 27.95 \times 10^{-3}\ mm$$

$C = 11400\ N/mm^2$ from table

$$V = \frac{\pi d_3 n_3}{60 \times 10^3} = 2.874\ m/s$$

$$P_d = \frac{21(2.874)(11400 \times 27.95 \times 10^{-3} \times 60 + 3479.47)}{21(2.874) + \sqrt{11400 \times 27.95 \times 10^{-3} \times 60 + 3479.47}}$$

$$= 6473.80\ N$$

$$P_{eff} = C_s P_t + P_d = 11693\ N.$$

$$FS = \frac{S_b}{P_{eff}} = \frac{25872}{11693} = 2.21$$

Design is satisfactory

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(iii) Fs on pitting $\underline{S_w = P_{eff} \times FS}$

$S_w = b\,Q\,d_p\,K$

$Q = \dfrac{2Z_g}{Z_g + Z_p} = 1.4783$

$K = 0.16\left(\dfrac{BHN}{100}\right)^2$

$S_w = 60\,(1.4783)\,(108)\,(0.16)\left(\dfrac{340}{100}\right)^2$

$= 17718.03\ N.$

$fs = \dfrac{S_w}{P_{eff}} = \dfrac{17718.03}{11693} = 1.52$

Design is safe

Example :-

A pair of spur gears with 20° full-depth involute teeth consists of a 19 teeth pinion meshing with a 40 teeth gear. The pinion is mounted on a crankshaft of 7.5 kW single cylinder diesel engine running at 1500 rpm. The driven shaft is connected to a two-stage compressor. Assume the service factor as 1.5. The pinion as well as the gear is made of steel 40C8 ($S_{ut} = 600\ N/mm^2$). The module and face width of the gears are 4 and 40 mm respectively.

(i) Using the velocity factor to account for the dynamic load, determine the factor of safety.

(ii) If the factor of safety is two for pitting failure, recommend surface hardness for the gears.

(iii) If the gears are machined to meet the specifications of Grade 8, determine the factor of safety for bending using Buckingham's equation.

(iv) Is the gear design satisfactory? If not, what is the method to satisfy the design conditions? How will you modify the design?

Given :- $P_{kW} = 7.5\ kW$, $\alpha = 20°$, $Z_p = 19$, $Z_g = 40$

$n_p = 1500\ rpm$ $\quad C_s = 1.5$, $S_{ut} = 600\ N/mm^2$ — pinion & gear

$m = 4$, $b = 40\ mm$.

(i) Both pinion & gears are made of same material so pinion is weaker

Y $n_p = 19 \rightarrow Y = 0.314$. $\quad \sigma_b = \frac{1}{3}(S_{ut}) = 200\ N/m^2$

$S_b = m b \sigma_b Y = 4(40)(200)(0.314) = 10048\ N.$

$S_b = P_{eff} \cdot f.s. \qquad f_s = S_b / P_{eff}$

Effective load P_{eff} :- $\quad V = \frac{\pi d_p n_p}{60 \times 10^3} = \frac{\pi (m Z_p) n_p}{60 \times 10^3} = 5.969\ m/s$

$P_{eff} = (C_s / C_v) P_t$. $\quad C_v = \frac{3}{3+v} = 0.3345$

$$P_t = \frac{2M_t}{d_p} = \frac{2}{m\, z_p} \times \frac{60\times10^6 P_{kw}}{2\pi\, n_p} = \frac{2(47746.48)}{m\, z_p} = 1256.49\ N.$$

$$P_{eff} = \frac{C_s}{C_v} P_t = \frac{1.5}{0.3345}(1256.49) = 5634.48\ N.$$

$$\boxed{FS = (S_b / P_{eff}) = 1.78}$$

(ii). $S_w = P_{eff}.\ FS. \Rightarrow\ b\, Q.\, d_p\, K = P_{eff}.\ FS.$

$$Q = \frac{2 Z_g}{Z_g + Z_p} = 1.356 \qquad K = 0.16\left(BHN/100\right)^2.$$

$$40 \times 1.356 \times m\, z_p \times 0.16\left(\frac{BHN}{100}\right)^2 = 5634.48 \times 2.$$

$$\boxed{BHN = 413.35}$$

(iii) FOS based on Buckinghams eqn. for Grade 8 gear.

for Grade 8 $\Rightarrow$ $e = 16 + 1.25\phi$ $\quad \phi = m + 0.25\sqrt{d}$

for pinion $e_p = 16 + 1.25(4 + 0.25\sqrt{4\times 19}) = 23.72\ \mu m$

for Gear $e_g = 16 + 1.25(4 + 0.25\sqrt{4\times40}) = 24.95\ \mu m$

$e = 23.72 + 24.95 = 48.67\ \mu m = 48.67 \times 10^{-3}\ mm$

$C = 11400\ N/mm^2 \longrightarrow$ from design data Books

$v = 5.969\ m/s$ $\qquad b = 40, \quad P_t = 1256.49\ N.$

$$P_d = \frac{21v\,(Ceb + P_t)}{21v + \sqrt{Ceb + P_t}} = 10555.17\ N.$$

$P_{eff} = (C_s P_t + P_d) = 12439.91\ N$

$S_b < P_{eff}$ & $S_w < P_{eff}$.

Design is unsatisfactory

iv) Design need to modify as FS is less than 1

Assume Grade 6 gear $\quad e = 8 + 0.63\phi$

for pinion. $\quad e = 8 + 0.63(4 + 0.25\sqrt{76}) = 11.893\ \mu m$

for Gear $\quad e = 8 + 0.63(4 + 0.25\sqrt{160}) = 12.512\ \mu m$

$$e = e_p + e_g = 24.405 \text{ μm}$$
$$= 24.405 \times 10^{-3} \text{ mm}$$

$$P_d = \frac{21V(Ceb + P_t)}{21V + \sqrt{Ceb + P_t}}$$

$$= 6560.53 \text{ N}$$

$$P_{eff} = (C_s P_t + P_d) = 1.5 \times 1256.49 + 6560.53$$
$$= 8445.265 \text{ N.}$$

Beam strength is lower than wear strength.

So $$fs = \frac{S_b}{P_{eff}} = \frac{10048}{8445.26} = 1.19$$

Design is satisfactory

Example:-

A pair of spur gears with 20° full depth involute teeth consists of a 20 teeth pinion meshing with a 41 teeth gear. The module is 3 mm while the face width is 40 mm. The material for pinion as well as for the gear is steel with an ultimate tensile strength of 600 N/mm^2. The gears are heat-treated to a surface hardness of 400 BHN. The pinion rotates at 1450 rpm and the service factor for the application is 1.75. Assume that velocity factor accounts for the dynamic load and the factor of safety is 1.5.

Determine the rated power that the gears can transmit

Example :-

The layout of a two stage gearbox is shown in figure. The no. of teeth on the gears are as follows.

$Z_1 = 20$, $Z_2 = 50$, $Z_3 = 20$, $Z_4 = 50$.

The pinion 1 rotates at 1440 rpm in anti-clockwise direction when observed from the left side of the page and transmits 10 kW power to the gear train. The pressure angle is 20. Draw a free body diagram of the gear tooth forces and determine the reactions at bearings E & F.